SIGMA POINT KALMAN FILTER FOR UNDERWATER TERRAIN-BASED NAVIGATION

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Abstract: Precise underwater navigation is crucial in a number of marine applications. Navigation of most autonomous underwater vehicles (AUVs) is based on inertial navigation. Such navigation systems drift off with time and external fixes are needed. This paper concentrates on one such method, namely terrain based navigation, where position fixes are found by comparing measurements with a prior map. Nonlinear Bayesian methods like point mass and particle filters are often used for this problem. Such methods are often computationally demanding. The sigma point Kalman filter (or unscented Kalman filter) is a nonlinear filter that does not resort to local linearizations, estimating the probability densities using a few deterministically chosen sigma points. The sigma point Kalman filter is not as computationally demanding as the aforementioned methods, and using real AUV data, the accuracies obtained are comparable to those of the point mass and particle filters.

1. INTRODUCTION

Precise underwater navigation is crucial in a number of marine applications. This paper focuses on navigation of autonomous underwater vehicles (AUVs), although the techniques described can also be used by other types of underwater vehicles, like submarines and remotely operated vehicles (ROVs).

Most AUV navigation systems are based on inertial navigation, see e.g. (Jalving et al., 2003). Inertial navigation systems drift off with time, even when velocity aiding is used. In order to allow extensive submerged operations, additional position fixes are needed. As GPS is not available underwater, one is often dependent on acoustic aiding of the vehicle, either from a mother ship or by using underwater acoustic transponders. In order to increase the autonomy of the vehicle and avoid costly pre-deployment of underwater transponders, terrain-based navigation is a favorable alternative. As an AUV in many cases carries a bathymetric sensor, it is natural to utilize bathymetric information in the navigation of the vehicle. For the methods presented here it is assumed that a prior bathymetric map database of the area exists. A multibeam echo sounder (MBE) is used as bathymetric sensor. MBEs are very well suited for terrain-based navigation, as a large area behind the vehicle is covered by the sensor.

Terrain-based navigation has been used for decades in aircraft and cruise missiles (Hostetler, 1978; Bergman, 1999), but for underwater vehicles the technique is rather new, though some papers have been published over the last few years, e.g. (Nygren and Jansson, 2004; Jalving et al., 2004; Ånonsen and Hallingstad, 2006). A variety of different terrain-based navigation methods have been proposed in the literature. Among the more sophisticated are the Bayesian methods, in which the position of the vehicle is estimated using a state-space model. Due to the strong nonlinearity of the measurements, Kalman filter-based methods

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have not proven to be suitable in most cases. Instead nonlinear Bayesian methods like point mass (PMF) and particle filters (PF) have been successfully applied to underwater navigation (Bergman, 1999; Jalving et al., 2004).

Due to the high computational demands of the PMF and PF, one must resort to a low-dimensional state-space model, typically estimating horizontal position and possibly depth bias only. Depth bias estimation was discussed in (Ånonsen and Hallingstad, 2006). The sigma point Kalman filter (SPKF), also known as the unscented Kalman filter (UKF), was first presented in (Julier and Uhlmann, 1997) and further discussed in (Wan and Van Der Merwe, 2000). It is based on the unscented transform, which estimates the underlying probability density using a set of deterministically chosen sigma points,which are propagated through the nonlinearities of the problem. The SPFK is considerably less computationally demanding than the PMF and PF. This also makes it possible to use a higher dimensional, more sophisticated filter model.

The SPKF was used for terrain referenced navigation in avionics applications in (Metzger et al., 2005). In (Lang, 2006), the SPKF for underwater terrain navigation was discussed, using simulated data. To the best knowledge of the authors terrain navigation results from the SPKF using real AUV data have not been published before.

The outline of this paper is as follows. First, the terrain navigation problem is presented, together with a short description of the PF and the SPKF. Second, computational results from real AUV data, using a Kongsberg Maritime HUGIN vehicle equipped with a multi-beam echo sounder, are presented, and the results from the SPKF and PF are compared. Finally, some conclusions are drawn and suggestions for further work are made.

2. THE ESTIMATION METHODS

2.1 Filter Model

Let $\boldsymbol{x}_k$ denote the horizontal position of the AUV at time step k, and $\delta\boldsymbol{x}_k = \boldsymbol{x}_k - \tilde{\boldsymbol{x}}_k$ the offset from the INS estimated position, $\tilde{\boldsymbol{x}}_k$. In (Ånonsen and Hallingstad, 2006), the following three-dimensional filter model was derived:

$$\delta\boldsymbol{x}_{k+1} = \delta\boldsymbol{x}_k + \boldsymbol{v}_k, \tag{1}$$

$$b_{k+1} = b_k, \tag{2}$$

$$\boldsymbol{z}_k = \boldsymbol{h}(\tilde{\boldsymbol{x}}_k + \delta\boldsymbol{x}_k) + \boldsymbol{I}\cdot b_k + \boldsymbol{w}_k, \tag{3}$$

where b_k denotes a possible depth bias term and $\boldsymbol{I}$ is the identity matrix. The highly nonlinear function $\boldsymbol{h}$ gives the sea depth at a given position according to a bilinear interpolation of a gridded depth map. The noises $\boldsymbol{v}_k$ and $\boldsymbol{w}_k$ are assumed to be white.

Using Bayes' formula, see e.g. (Bar-Shalom et al., 2001), the optimal minimum mean square error estimator of this model can be derived. However, this solution contains integrals that are analytically unsolvable and need to be approximated. The PF and SPKF are examples of such approximation methods.

2.2 The Bayesian Bootstrap Particle Filter

Particle filtering has gained attention the recent years for estimation in nonlinear systems, in areas such as signal processing, tracking and navigation, to name a few. Introductions to the principles of particle filtering can be found in (Arulampalam et al., 2002; Doucet et al., 2001). The results in this paper are obtained using a Bayesian bootstrap particle filter, which is one of the simplest particle filter algorithms, first presented in (Gordon et al., 1993).

The principle of the algorithm is rather simple. First a particle set is sampled from the initial distribution, such that the density of of particles in an area reflects the probability that the vehicle is in that area. Then, at each time step, the particles are propagated through the filter process equations (1)–(2). Each particle in the set is assigned an importance weight proportional to the likelihood $p(\boldsymbol{z}_k|\boldsymbol{x}_k)$ of the measurements, and the particles are then resampled with replacement according to their importance weight. This completes one cycle of the algorithm. The resampling step is crucial in order to avoid degeneration of the particle set. The systematic resampling algorithm from (Arulampalam et al., 2002) is used.

The reason for choosing the relatively simple Bayesian bootstrap filter is that it has proven to yield good results in this application. Other, more sophisticated particle filters, like the Sequential Importance Resampling (SIS) algorithm with effective particle number estimation (Doucet et al., 2001) have also been tested on the same data, without improving the results significantly.

2.3 The Sigma Point Kalman Filter

In the SPKF (Julier and Uhlmann, 1997, 2004) the underlying distributions are approximated using a set of sigma points. Contrary to the PF particle cloud, the sigma points are chosen deterministically, based on knowledge of the probability distributions of the problem. The sigma points are propagated through the nonlinearities of the process and measurement equations (1)–(3) to yield a new sigma point approximation of the posterior density. For the SPKF process and measurement update equations, see (Julier and Uhlmann, 2004).

In this particular estimation problem the SPFK equations are simplified, since the process model is linear. The time update can therefore be done using the conventional Kalman filter time update equations. Consequently, the unscented transform is used for the measurement update only. Gaussian assumptions are made, and the sigma points are selected using the the symmetric $2n_x + 1$ selection scheme described in (Julier and Uhlmann, 2004).

3. COMPUTATIONAL RESULTS

The algorithms were tested using data from a Kongsberg Maritime HUGIN AUV equipped with an EM3000 multi-beam echo sounder (MBE). The data were collected in November 2001 in an area in the Oslo fjord. A gridded bathymetric map of the area with a horizontal resolution of 10 m was used in the algorithms. In order to test the convergence properties of the algorithms they were initialized with an error of 50 m in each horizontal direction. The results were compared to a post-processed trajectory, using measurements from the real-time navigation system, as well as acoustic aiding from the mother ship. This trajectory has an accuracy of approximately 1 m (1σ).

Fig. 1, 2 and 3 show horizontal errors from the SPKF and the PF (using 1000 particles) in three different areas of the test run. The results in Fig. 1 and 2 are from rather rough areas, well suited for terrain navigation, whereas the results in Fig. 3 are from a flat area. As can be seen in Fig. 1 and 2, the results from the SPKF are relatively accurate in the well suited terrain. In Fig. 1 the SPKF is much less accurate than the PF, with a final horizontal error of around 22 m, compared to around 6 m for the PF. However, the typical behavior is more like that in Fig. 2, where the accuracy of SPFK is comparable to that of the PF. In the flat area, neither method works satisfactorily, as expected. The performance in a flat area is shown in Fig. 3, where large errors are seen for both methods.

As reported in (Ånonsen and Hallingstad, 2006), problems with overconfidence occur in Bayesian terrain navigation. However, these problems can be countered for to some extent by sub-sampling the MBE beams. The covariances obtained from the SPKF are very similiar to those from the PF, both in the rough and flat terrain.

4. CONCLUSIONS

This paper has presented results from terrain aided underwater navigation using an SPKF on real AUV data. The results are generally quite good, though the overall position accuracy is slightly poorer than that obtained with a PF. However, the SPFK algorithm is computationally less complex than the PF, which makes it easier to extend the state vector to more than three dimensions. A higher dimensional state vector is crucial if one wants to develop a more realistic filter model.

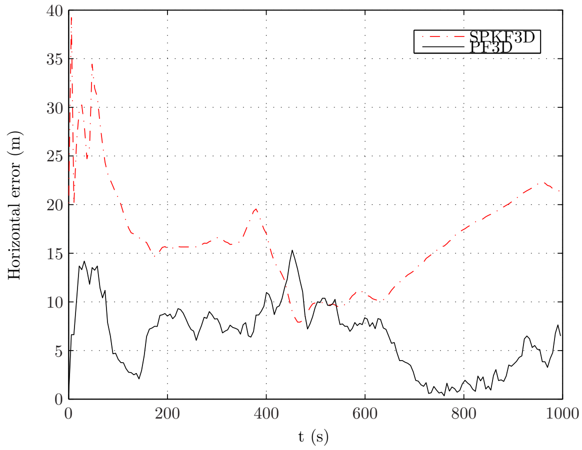

Fig. 1. Comparison of SPFK and PF 1000–2000 s from start.

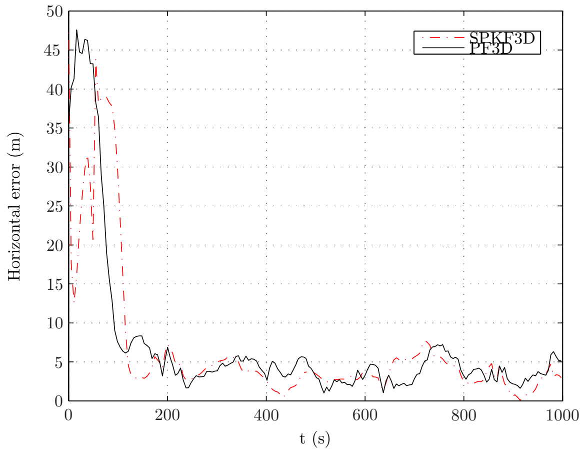

Fig. 2. Comparison of SPFK and PF 3000–4000 s from start.

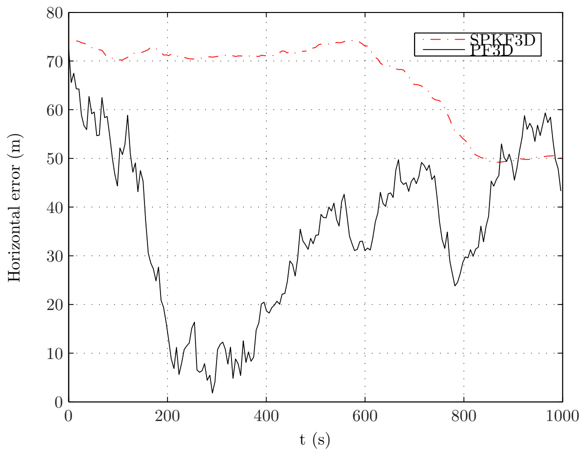

Fig. 3. Comparison of SPFK and PF 9000–10000 s from start.

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