

Underwater Transponder Positioning and Navigation of Autonomous Underwater Vehicles

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Abstract—Navigation of underwater vehicles has been and remains a substantial challenge to all platforms. The need for improved accuracy and robustness, sustainability, and de-risking develops with the emergence of new applications, and with the growing acceptance of autonomous underwater vehicles (AUVs) in both military and civilian institutions. One of the main driving factors is the ability to carry out long-duration missions fully autonomous and without supervision from a surface ship. Combined with inertial navigation, the use of one or several transponders on the seabed is an accurate and cost-effective approach toward solving several of these challenges. The principle discussed in this paper is called underwater transponder positioning (UTP), and requires only one transponder due to tight coupling with the inertial navigation system (INS). For many scenarios UTP may be a better alternative than using a long baseline (LBL) system.

This paper reports in-situ and post-processed navigation results obtained with a state-of-the-art UTP aided INS, onboard a HUGIN 1000 AUV. The results demonstrate the feasibility of UTP in large-scale autonomous operations. Excellent real-time navigation is achieved, and the accuracy obtained in post-processing is shown to be close to that obtained when aiding the INS with an ultra-short baseline (USBL) positioning system.

I. INTRODUCTION

After two decades of dedicated research and development, autonomous underwater vehicles (AUVs) are today becoming accepted by an increasing number of users in both military and civilian institutions. The number of AUV systems sold worldwide is well into triple digits. The bulk of these systems have been manufactured within the last five years, so the sector is in rapid growth. Next to improved payload quality and endurance, much of this success is due to recent advances in navigation sensor technologies and fusion algorithms [1], [2], [3]. Despite these achievements, navigation remains a substantial challenge to all submersibles. The actual autonomy of the vehicles in existence today is also limited. Further advances in both areas will enable new operations which earlier have been considered infeasible, or at best difficult.

This paper is concerned with inertial navigation of AUVs, with particular focus on tight integration of range measurements from one or several transponders. The concept, called underwater transponder positioning (UTP), is available to all the Kongsberg Maritime HUGIN AUVs. The first at-sea demonstration of single transponder UTP aided inertial navigation was carried out in 2003, as described in [2]. This paper is a continuation to this work, incorporating multiple transponders, deployed far apart. An overview of additional

inertial navigation system (INS) aiding tools available to HUGIN and other AUVs is found in [3]. While the use of transponders is not completely self-governing (requires deployment), it allows for truly autonomous operations once deployed on the seabed (upcoming transponders have a battery capacity of five years). Typical applications of UTP include:

- ◇ Pipeline inspection and intervention.
- ◇ Under ice surveys (transponders may be deployed from an ice breaker or along the ice ridge).
- ◇ Scenarios where repeated dives in an area are required.
- ◇ Autonomous surveys where surfacing or support from a surface ship are infeasible due to e.g. heave traffic.

UTP is also complementary to traditional ultra-short baseline (USBL) and long baseline (LBL) positioning. Compared to an LBL system, UTP has improved accuracy due to tight coupling with the INS, increased operating area and significantly less deployment cost, since only one transponder is required.

Data from a HUGIN 1000 AUV mission prove the in-situ real-time navigation performance of the UTP aided INS. HUGIN navigated autonomously with UTP as the only position aiding for roughly 8 h, much of the time without. The data analysis also include a comparison with conventional USBL aided INS, as well as results showing the accuracy enhancement obtained by using NavLab [4] in post-processing.

For the remainder, UTP-INS is used for short when discussing UTP aided INS without distinguishing on accompanying sensors. Additional notation is appended when discussing the integration of specific aiding sensors such as USBL and Doppler velocity log (DVL). Pressure sensor data are always present. This paper is furthermore organized as follows. The remainder of this section reviews inertial navigation of underwater vehicles. Section II describes some of the principles of UTP, as well as operational procedures. The experimental setup is described in Section III, followed by an experimental evaluation of the proposed navigation system in Section IV.

A. Underwater Vehicle Inertial Navigation

An INS calculates position, velocity and attitude using high frequency data from an inertial measurement unit (IMU) which typically consists of three accelerometers measuring specific force and three gyros measuring angular rate, all relative to the inertial space. Due to inherent errors in the gyros and accelerometers, the solution of the navigation equations

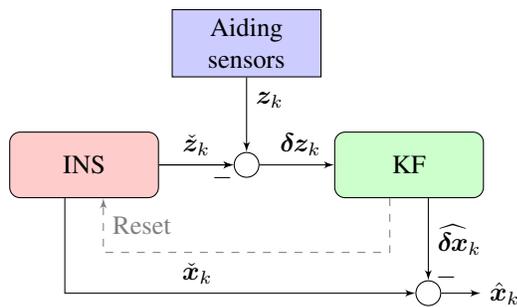


Fig. 1. Outline of conventional aided INS.

embedded in the INS will have an unbounded drift unless counteracted. A performance measure for an INS is given by its pure inertial drift in position, where the divergence rate depends on the IMU quality. A navigation grade INS drifts in the order of one nautical mile per hour. Since an INS is a diverging system, an aiding framework is needed to limit or reduce the error growth. An overview of INS aiding tools is given in [2], [3]. For autonomous missions it may be important to retain good navigation accuracy between position updates, which will usually be sparse. The use of bottom-track data from a DVL is today the most common approach. See Section I-B for additional information regarding DVL-INS.

In order to fuse the data from the INS and the aiding sensors, some form of filtering must be implemented. This is typically accomplished using a Kalman filter (KF). An outline of a conventional aided INS is shown in Fig. 1, where the KF input is taken as the difference between the output from the appropriate aiding sensors and the INS. A perturbation method is used in this paper for deriving the INS error states. The states are included in the KF with the assumptions of small errors, i.e. first order approximation. The KF also estimates the colored errors of the navigation sensors. The INS and aiding sensors considered in this paper are shown in Fig. 2.

B. DVL-INS

In many practical situations position measurements will be unavailable for extended periods of time and the INS will then chiefly depend on external velocity aiding. While alternatives exist (see e.g. [5], [6], [7]), the application of DVL with bottom-track is predominant. If within sensor range, the DVL measures the vehicle linear velocity relative to the seabed along four acoustic beams. Data obtained from a minimum of three beams are combined in order to calculate the velocity.

The amount of literature on error sources in DVL based navigation is extensive. For DVL-INS the horizontal position drift is determined by the error in the estimated Earth-fixed velocity. The main contributors are body-fixed velocity error, and heading error. The error in estimated body-fixed velocity is mainly determined by the low-frequency errors of the DVL itself (e.g. alignment and speed of sound scaling). These errors are not observable if the vehicle is traveling along a straight line and without position aiding. High frequency velocity errors are on the other hand estimated by means of the IMU. As for the error in heading, it is determined by

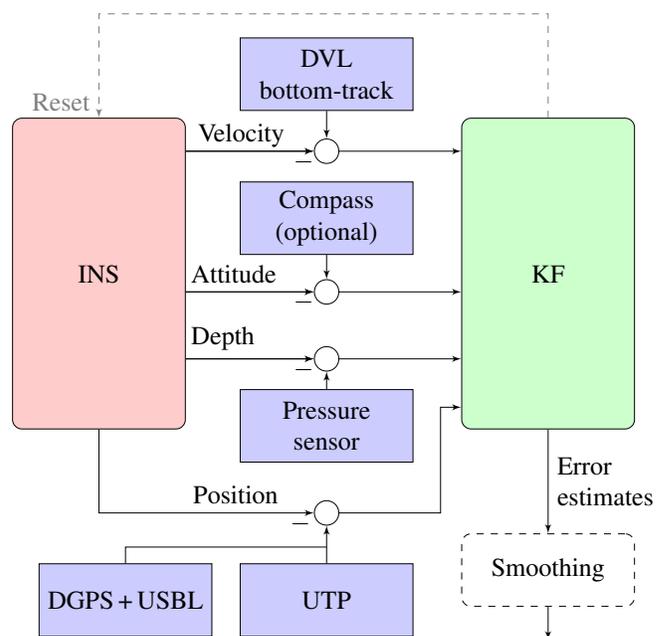


Fig. 2. The integrated INS and aiding sensors considered in this paper. A large toolbox of additional aiding sources is available to the HUGIN AUVs.

the gyrocompassing capability of the integrated system. The heading estimation error will typically be of low frequency, corresponding to non-observable gyro bias dynamics.

In order to minimize the drift in the DVL-INS, the heading estimate should be properly initialized prior to launch or before carrying out an autonomous mission with sparse position aiding. It is also vital that the misalignment between the DVL and the IMU is compensated for. Sound speed scaling may be reduced by using an external CTD sensor. As mentioned, a KF can also compensate for part of the DVL errors when running more complex survey patterns, or when position updates are available. The reader is referred to [2] for a further discussion.

Examples of in-situ DVL-INS accuracy obtained by the HUGIN 1000 AUVs are shown in Fig. 3. The real-time navigation system onboard HUGIN is called NavP (navigation processor). In each of the dives HUGIN ran along two straight lines in opposite direction, each roughly 7 km in length. Similar verification trials prove a NavP navigation accuracy in the order of 0.1% of distance traveled (or better) when running without position aiding along a straight line. Depending on the application, a low in-situ DVL-INS drift may be imperative to mission success. A low real-time drift is also important when utilizing the UTP range measurements and when traveling between the transponders, as discussed in Section II. Note also that the accuracy and robustness may be further enhanced in NavLab post-processing (see Section III-C for further details).

II. UNDERWATER TRANSPONDER POSITIONING

LBL and USBL acoustic positioning are both well known principles which today are used routinely in a number of applications, including underwater vehicle INS aiding. When operating within an LBL network the vehicle interrogates

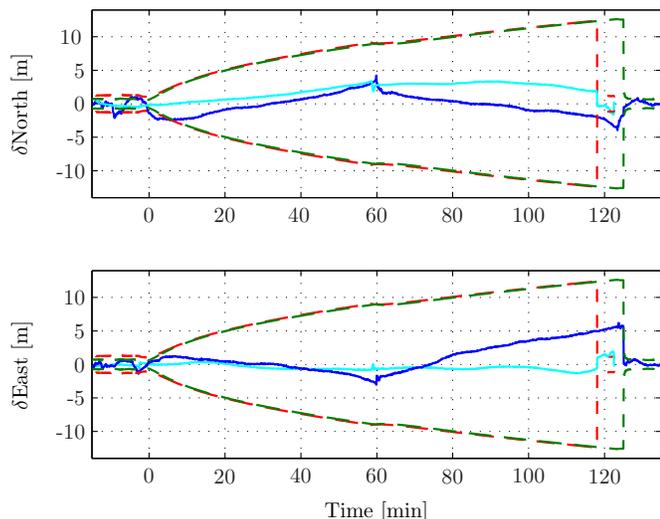


Fig. 3. Proven in-situ navigation performance of DVL-INS on HUGIN 1000 AUVs when traveling along two reciprocal lines, each about 7 km. The blue and cyan (solid) data show the north and east position errors from two dives. The KF real-time uncertainties (1σ) are shown in red and green (dashed). The positioning aiding was turned off at time 0 and remained off for about 2 h (topside logging of DGPS-USBL position measurements continued at 1/3 Hz).

the transponders, and the replies are used for calculating the range to each them. If the geographical position of each transponder is known, a unique position can be computed by triangulation (three or more transponders are needed if AUV and transponder depths are known). As for USBL, a typical approach is to measure the range and bearing of a transponder on the underwater vehicle relative to a transducer mounted on a surface vessel. This is e.g. the case when using Kongsberg Maritime HiPAP together with the HUGIN AUVs. A global position measurement, which may be transmitted to the submersible using an acoustic link, can be obtained by combining surface ship GPS and USBL measurements.

Within the last decade, an increasing number of single transponder systems have been proposed as alternatives to LBL and USBL. The growing interest is in large part due to the significant logistics and calibration involved when establishing an LBL network. Also, following an AUV with surface ship USBL may not always be feasible. On the technical side, the usage of single transponder navigation (range aiding) has been made possible due to improved dead-reckoning capabilities, e.g. navigation accuracy of DVL-INS. Further details on single transponder range aiding and UTP is given subsequently.

Common to the systems above is the dependency on two-way travel time (TWTT). An alternative in single-range navigation is to explore synchronous-clocks for direct measurement of one-way travel time (OWWT). See e.g. [8] for details.

A. UTP-INS

From a navigation point of view a single range transponder may be thought of as an underwater lighthouse providing the AUV with ranges relative to its fixed geographical location. By fusing this information, a global position is obtained which may be used for aiding the INS. In UTP this is

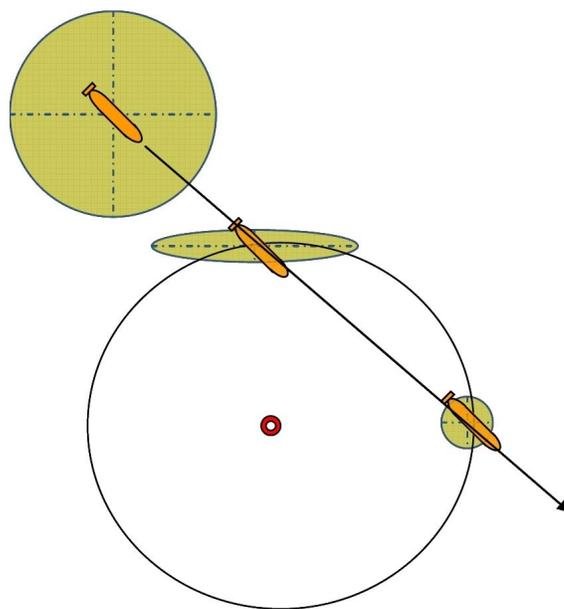


Fig. 4. Principle and effectiveness of UTP-INS. The shaded areas are examples of horizontal covariance ellipses, initially compressed radially.

done in tight integration with the INS. In contrast, most LBL aiding schemes are loosely coupled. In-depth simulations (including full acoustic modeling, ray-tracing, line-of-sight considerations) were carried out in [9] for pipeline surveying and touchdown monitoring on the Ormen Lange gas-field in the Norwegian Sea. For the terrain and environment considered in the report, UTP aided INS was found to be better than LBL aided INS in all aspects since each individual range measurement in UTP is optimally utilized to provide increased accuracy and robustness toward loss of transponder coverage.

Single transponder navigation is not a new concept. As mentioned, the first at-sea demonstration of single transponder UTP aided inertial navigation was carried out in 2003, as described in [2]. This paper is a continuation to this work, incorporating multiple transponders, deployed far apart. Work by other authors on single range navigation include [10], [11], [12], [13], [14], [15], [16], [17]. The majority of the work rely on either least-squares (LS) or Kalman filtering in order to calculate the vehicle position based on one or several ranges.

In principle a range measurement only tells that the vehicle is located somewhere on a circle with the transponder in its center. An innovative algorithm is implemented in UTP for determining the best possible location on the circle and for tightly integrating the range measurements with the INS. As the vehicle passes through the transponder area the algorithm takes advantage of the slow error drift of the DVL-INS (see Section I-B) and the geometry change due to physical vehicle movement. The principle behind UTP is illustrated in Fig. 4.

1) *Transponder Deployment and Georeferencing:* An example showing two Kongsberg Maritime UTP transponders is shown in Fig. 5. The transponders are equipped with a release mechanism for easy recovery. As for deploying the UTP transponders, standard network design parameters such as



Fig. 5. Two Kongsberg Maritime UTP transponders ready for deployment. Dissolvable sandbags used as weights in this particular mission. The new generation transponders have a battery life capacity up to five years.

maximum range and distance between consecutive transponders (depending on DVL-INS performance and required survey accuracy) must be considered. Also, in order to minimize errors related to speed of sound inaccuracy, the transponder depth should be roughly similar to the operational depth of the AUV. In practice the speed of sound can be measured by a CTD on the vehicle or on the UTP transponder (or both).

For ranging techniques like UTP to work, the geographical location of the transponders must be known or estimated (box-in). The preferred method is to measure the position directly using USBL on a surface ship. Using Kongsberg Maritime HiPAP and assuming a GPS north and east accuracy of 0.4 m (1σ), the Earth-fixed location of the transponder can be determined to within 0.6, 1, 2 and 3 m at 500, 1000, 2000 and 3000 m depth, respectively. As the depth decreases the GPS accuracy becomes the dominant source in the error budget. An important observation is that the box-in process will usually be done at the same time as deploying the transponders, hence the additional operational time needed is small. Other box-in methods also exist. A simultaneous localization and mapping (SLAM) approach is e.g. applied in [17] in order to obtain a successively improving estimate as the AUV interrogates the transponder. A limitation of this approach is that the box-in accuracy is lower bounded by the accuracy of the aided INS at the time the box-in process starts, hence making it less suitable to deep water operations (e.g. involving operations above the DVL range) or when transponders are deployed far apart.

2) *Operational Procedures*: The usage of UTP transponders is not completely self-governing since it requires deployment and box-in. It does however allow for truly autonomous operations once on the seabed, and for a long time due to high battery capacity. The AUV may navigate autonomously with bounded error by visiting the network occasionally.

When running UTP with the HUGIN AUVs, the interrogation of the transponder may be started and stopped both manually and automatically. In Auto mode the interrogation is initiated when operating inside the transponder range, also taking the DVL-INS navigation uncertainty into account.



Fig. 6. HUGIN 1000 during recovery onboard H.U. Sverdrup II.

TABLE I
IMU SPECIFICATIONS

Model	Bias		Scale Factor		Rate
	Gyro	Acc	Gyro	Acc	
Honeywell HG9900	0.003 deg/h	25 μ g	5 PPM	100 PPM	300 Hz

TABLE II
PRIMARY NAVIGATION AIDING SENSORS

Variable	Sensor	Precision	Rate
Position	Kongsberg HiPAP USBL*	<20 cm, 0.12 deg	Varying [†]
	Kongsberg UTP	<10 cm	Varying [‡]
Velocity	RDI DVL 300kHz	$\pm 0.4\% \pm 0.2$ cm/s	>1 Hz
Pressure	Paroscientific	0.01 % full scale	1 Hz

* Surface ship GPS and USBL are combined to give a global vehicle position. The accuracy of the final position also depends on the ship GPS precision.

[†] Depends on the slant range. While submerged, the AUV receives position updates at about 1/30 Hz, from the surface via an acoustic commando link.

[‡] Depends on the range from the AUV to the transponder. Usually >1/2 Hz.

III. EXPERIMENTAL SETUP

An overview of the experimental setup, including vehicle particulars, employed navigation sensors, mission trajectory, and the processing of raw navigation data, is given in this section. The experimental results are discussed in Section IV.

A. Vehicle Description

The performance and comparison of the integrated INS with and without UTP aiding is evaluated using in-situ (real-time) NavP navigation data and raw sensor data. The data were collected by a HUGIN 1000 AUV with 3000 m depth rating, owned and operated by the Norwegian Defence Research Establishment (FFI). The launch and recovery system and the AUV are shown in Fig. 6. The diameter and length (base version) of the vehicle are 0.75 and 5.3 m. It can operate with full payload for 25 h at a cruising speed of about 2 m/s.

HUGIN 1000 is equipped with an aided INS, as outlined in Fig. 2. Additional aiding tools are also available, but are not discussed any further in this paper. Some IMU specifications are shown in Table I. The primary navigation aiding sensors relevant to the data utilized in this work are listed in Table II.

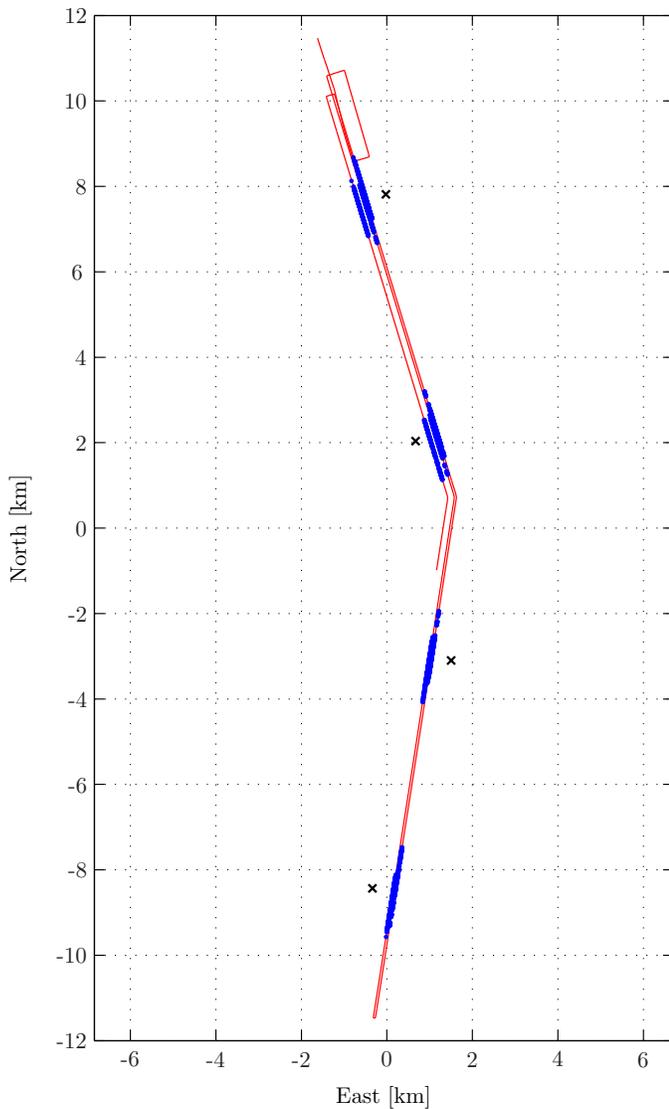


Fig. 7. UTP-INS navigation - August 2008. The true AUV trajectory is shown in red (solid), the positions derived from the range measurements are shown in blue (●), and the UTP transponder locations are shown in black (x).

B. Experiment Description

The data in this paper were collected August 2008 in the northern parts of Norway, close to Tromsø. The vehicle trajectory and transponder locations are shown in Fig. 7. The mission is representative for a pipeline inspection survey.

HUGIN navigated in real-time (NavP) with UTP as the only position aiding tool for roughly 8 h, much of the time without. The AUV occasionally revisited the four UTP transponders, which were deployed about 6 km apart. The transponder depths ranged from 270 to 325 m, and the vehicle height above the seabed was about 15 m throughout the mission. A 30 kHz system was used, which in this mission gave up to 1.2 km practical range for each transponder, as seen in Fig. 8. Lower frequency yields longer range (at the cost of precision). Due to shadow from the AUV hull, the first received ranges appeared at about 600-700 m when approaching the transponders.

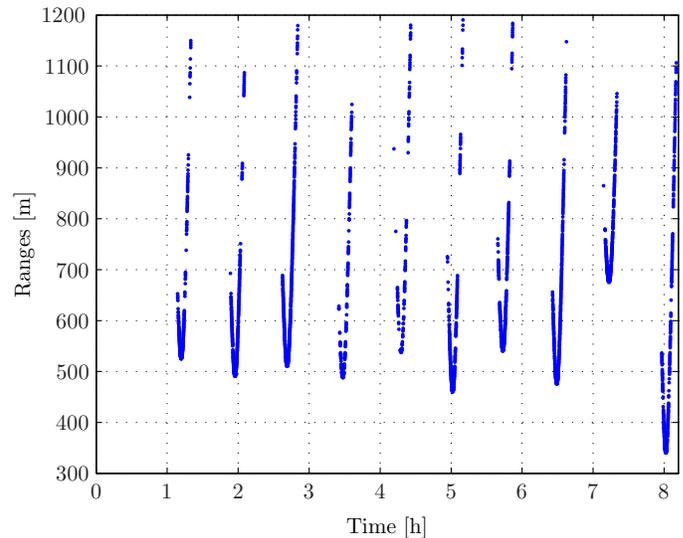


Fig. 8. Practical ranges obtained for the survey trajectory and transponder locations in Fig. 7. A total of 10 passes were done, and ranges up to 1200 m were obtained. When approaching the transponders the first ranges were received later than desirable, and is due to the shadow from the hull. An alternative location of the vehicle mounted transducer is being considered.

C. Data Post-Processing

The real-time solution from NavP is the original navigation data collected at sea. As for the post-processed results, raw navigation sensor values are used throughout. The re-navigation routines are implemented in NavLab [4], a tool which has been used extensively with all the HUGIN AUVs since the late 1990's. In addition to re-navigating the real-time navigation system, NavLab also contains offline smoothing functionality, based on a Rauch-Tung-Striebel (RTS) implementation. The RTS smoother utilizes both past and future sensor measurements and KF covariances, hence efficiently improving the integrity and accuracy of the final navigation solution [18]. In this paper the smoothed USBL-DVL-INS solution with the highest navigation sensor update rates available serves as the reference when evaluating the performance of the UTP-INS. DVL data at 3 Hz and USBL data at about 1/2 Hz were utilized. The accuracy of the RTS smoothed reference position was estimated to be 0.7 m (1σ) in north and east.

IV. EXPERIMENTAL RESULTS

This section evaluates the performance of the UTP-INS described in Section II-A. As mentioned above, the RTS smoothed reference solution serves as the ground truth during comparison. All the results have also been verified by comparing the AUV multi-beam echosounder (MBE) and side-scan (SSS) data with data collected using the surface ship Kongsberg Maritime EM710. Both in-situ real-time results and post-processed results are investigated. An estimation error is taken as the difference between the RTS reference solution and the navigation solution being evaluated.

The in-situ navigation accuracy of NavP is shown in Fig. 9. The maximum horizontal error when running UTP-DVL-INS (completely autonomous) is about 8 m, but for most parts

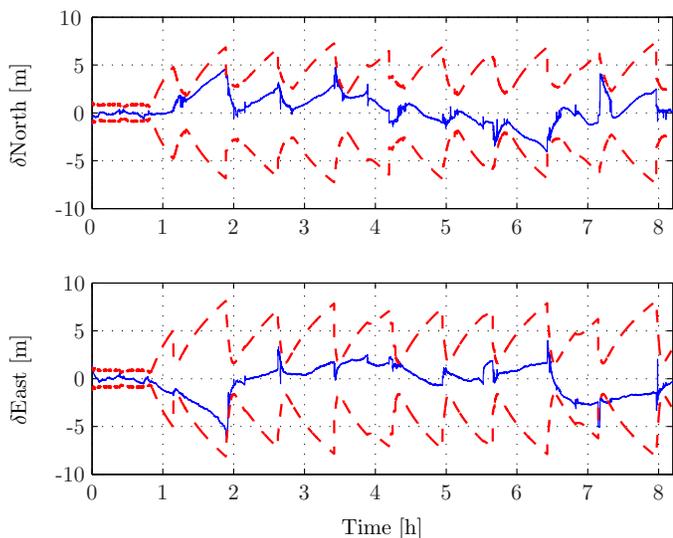


Fig. 9. The blue (solid) data show the north and east position errors of the in-situ UTP-DVL-INS. Kongsberg HiPAP USBL was used during the first 45 min. The real-time uncertainties (1σ) are shown in red (dashed).

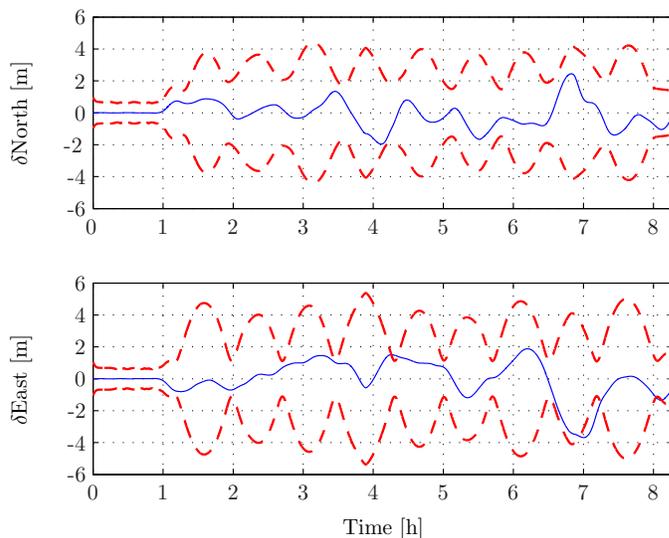


Fig. 11. The blue (solid) data show the north and east position errors of the NavLab post-processed UTP-DVL-INS. Kongsberg HiPAP USBL was used during the first 45 min. The uncertainties (1σ) are shown in red (dashed).

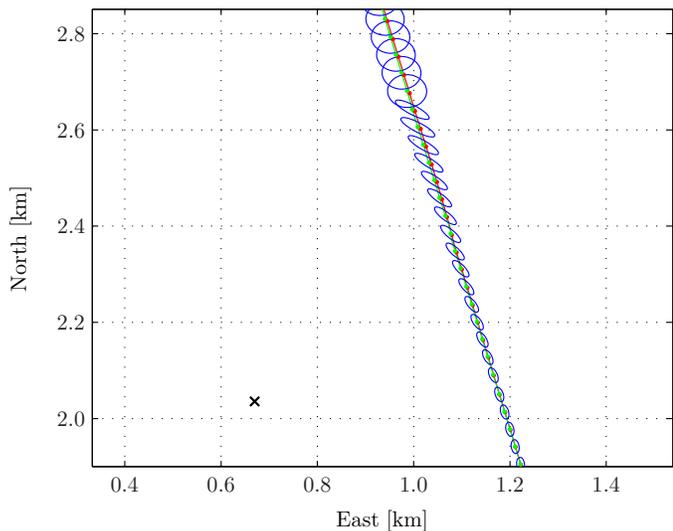


Fig. 10. The true AUV trajectory is shown in red (solid), the trajectory estimated by NavP in real-time is shown in green (solid), and the UTP transponder location is shown in black (x). The axes are the same as in Fig. 7. The blue circles and ellipses are the horizontal covariance matrices, scaled to 5σ for easier display. As seen in Fig. 9, the errors are within 1σ throughout.

within 5 m. The sample mean and standard deviation of the errors are 0.5 ± 1.6 m in north and -0.3 ± 1.6 m in east. During the first 45 min in Fig. 9 USBL aiding was used regularly, hence lowering the uncertainty and error. The use of USBL was mainly done for control purposes and for avoiding any initial position error prior to running autonomously. As for INS initialization, this was done while on deck using GPS and during the very start with USBL and DVL. The mission was carried out north of the polar circle, hence requiring longer time for initialization. The real-time heading uncertainty in NavP was in the order of 0.07 deg at launch and about 0.05 deg when starting the 8 h part running with UTP.

The NavP performance is also shown in Fig. 10 for a small subset of the data. The NavP position covariances are visualized as error ellipses (5σ is used for easier display). As seen in Fig. 9, the north and east errors are well within 1σ most of the time. The same ellipses illustrate the principle and effectiveness of UTP-INS (see also Fig. 4). The error covariance is initially compressed radially, but is also compressed tangentially as the vehicle travels through the UTP zone.

As mentioned in Section III-C it is also possible to further enhance the navigation accuracy by using NavLab. NavLab is particularly effective when position measurements are sparse, as is the case in this mission. The post-processed navigation accuracy is shown in Fig. 11. The maximum horizontal error when running UTP-DVL-INS is now about 5 m, but for most parts within 3 m. The sample mean and standard deviation of the errors are 0.08 ± 0.8 m in north and 0.04 ± 1.14 m in east. The errors are again consistent and within 1σ most of the time. The accuracy obtained in NavLab post-processing is close to the accuracy of the reference solution where USBL was used.

V. CONCLUSION

This paper has reported the usage of underwater transponder positioning (UTP) and the tight integration with INS. Both in-situ and post-processed navigation results verify that UTP aiding is a feasible and accurate approach for large-scale operations, improving underwater navigation capabilities for systems where the need for flexibility, redundancy and autonomy is important. A strength of UTP is that only one transponder is needed to effectively bind the INS error drift.

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